Bonus Question:
How Does Flexible Incentive Pay Affect Wage Rigidity?

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Motivation

- Sluggish wage adjustment over the business cycle is important in macro
  - Unemployment dynamics (Hall 2005, Hagedorn & Manovskii 2008, Gertler & Trigari 2009)
  - Inflation dynamics (Christiano et al 2005, 2016)

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- One challenge for models w/ wage rigidity: incentive pay
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- **This paper:** *how does flexible incentive pay affect wage rigidity?*
  - Incentive pay: piece-rates, bonuses, commissions, stock options or profit sharing
  - 30-50% of US workers get incentive pay (Lemieux, McLeod and Parent, 2009; Makridis & Gittelman 2021)
  - Including 25-30% of low wage workers

Wage Cyclicality from Incentives Does Not Mute Unemployment Response

This paper: incentive pay + unemployment dynamics + slope of price Phillips Curve

- Flexible incentive pay = dynamic incentive contract with moral hazard (Holmstrom 1979; Sannikov 2008)
- Unemployment = standard labor search model (Mortensen & Pissarides 1994)
- Phillips Curve: sticky price model with labor search (Blanchard & Gali 2010, Christiano et al. 2016)
- Allows flexible + cyclical incentive pay and long-term contracts consistent with microdata

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Result #1: Wage cyclicality from incentives does not dampen unemployment responses

Unemployment dynamics first-order identical in two economies calibrated to same steady state:

1. Economy #1: labor search model with flexible incentive pay + take-it-or-leave-it offers
2. Economy #2: labor search model with perfectly rigid wages as in Hall (2005)

Intuition: lower incentive pay raises profits, but worse incentives reduces effort + lowers profits

- **Optimal contract:** effect of wage + effort on profits cancel out

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Result #2: Wage cyclicality from incentives does not affect slope of price Phillips Curve

- **Optimal contract:** Effort movements ensure effective **marginal costs are rigid**

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Result #3: Calibrated model: $\approx$ 45% of wage cyclicality **due to incentives**, remainder due to bargaining

→ Calibrate simple models without incentive pay to wage cyclicality that is 45% lower than raw data

- More empirical work should separately measure wage cyclicality due to **incentives vs bargaining**

▸ Literature

Static Model

Dynamic Model

Numerical Exercise

Conclusion

Roadmap

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   - Equivalence result for unemployment responses
2. **Introduce sticky prices**
   - Equivalence result for slope of Phillips Curve
3. **Introduce non-incentive wage cyclicality**
   - Bargaining/outside option fluctuations
   - Non-incentive wage cyclicality **does** affect marginal costs

Frictional labor markets

- Measure 1 of workers begin unemployed and search for jobs; remain unemployed if unmatched
- Firms post vacancies v at cost κ to recruit workers
- Vacancy-filling rate is $q(\theta) \equiv \Psi\theta^{-\nu}$ for $\theta \equiv v/u$ market tightness

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Workers' preferences

- Workers derive utility from consumption c and labor effort a with utility $u(c, a)$
- Employed workers consume wage w and supply effort a
- Unemployed workers have value $U \equiv u(b, 0)$

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Technology

- Firm-worker match produces output $y = z(a + \eta)$
  - z: aggregate labor productivity, always common knowledge
  - η: i.i.d., mean zero output shock with distribution $\pi(\eta)$
- Firms pay workers wage w, earn expected profits from a filled vacancy:

$$J(z) = \mathbb{E}_\eta \left[z(a + \eta) - w\right]$$

Employment Dynamics in Static Model

- Free entry to vacancy posting guarantees zero profits in expectation:

$$\kappa = \underbrace{q(v)}_{Pr\{\text{Vacancy Filled}\}} \cdot \underbrace{J(z)}_{\text{Value of Filled Vacancy}}$$

- Response of Employment to productivity z: ▸ Derivation

$$\boxed{\frac{d \log n}{d \log z} = constant + \left(\frac{1-\nu}{\nu}\right) \cdot \frac{d \log J(z)}{d \log z}}$$

- Next: solve for dJ/dz to determine employment responses

First Order Effect of Change in Labor Productivity z

Consider effect of small shock to z on expected profits $J(z)$:

$$\frac{dJ(z)}{dz} = \frac{d\mathbb{E}_\eta\left[z(a+\eta)-w\right]}{dz}$$
$$= \mathbb{E}_\eta\left[\underbrace{\frac{\partial[z(a+\eta)-w]}{\partial z}}_{\text{Direct Productivity}} + \underbrace{\frac{\partial[z(a+\eta)-w]}{\partial w}\cdot\frac{dw}{dz}}_{\text{Wages}} + \underbrace{\frac{\partial[z(a+\eta)-w]}{\partial a}\cdot\frac{da}{dz}}_{\text{Incentives}}\right]$$

If labor productivity shocks change effort, incentives can partially offset marginal cost effect

Next: different models of a and w

Two Models of a and w

$$\frac{dJ(z)}{dz} = \mathbb{E}\left[a - \frac{dw}{dz} + z\frac{da}{dz} \right]$$

Model	a	w	$\frac{dJ(z)}{dz}$
Fixed effort and wage (Hall 2005)			
Optimal incentive contract (Holmstrom 1979)			

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$$J(z) \equiv \max_{a(z),w(z,y)} \mathbb{E}[z(a(z)+\eta) - w(z,y)]$$

subject to

$$\text{incentive compatibility constraint:} \quad a(z) \in \arg\max_{\tilde{a}(z)} \mathbb{E}\left[u(w(z,y),\tilde{a}(z))\right]$$

$$\text{participation constraint w/ bargaining:} \quad \mathbb{E}\left[u(w(z,y),a(z))\right] \geq \mathcal{B}$$

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- Properties of the contract:
  1. Promised utility is constant $\mathcal{B}$ $\rightarrow$ all wage cyclicality due to incentives (relaxed later)
  2. Incentives vs insurance—pass through of y into w

Wage Cyclicality from Incentives Does Not Dampen Employment Response

$$\frac{dJ(z)}{dz} = \mathbb{E}\left[a + \overbrace{z\frac{da}{dz} - \frac{dw}{dz}}^{=0 \text{ by Envelope Thm}} \right]$$

Model	a	w	$\frac{dJ(z)}{dz}$
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$\Longrightarrow$ In both rigid wage and flexible incentive pay economies:

$$\frac{d\ln n}{d\ln z} = constant + \frac{1-\nu}{\nu} \cdot \frac{d\ln J(z)}{d\ln z}$$

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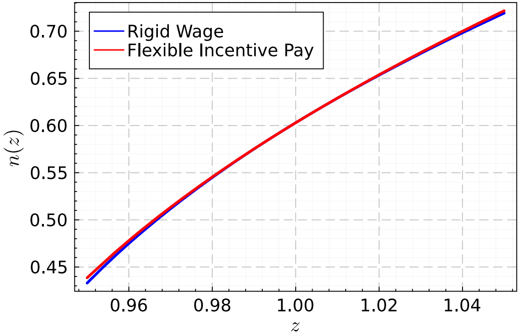
$\Longrightarrow$ In both rigid wage and flexible incentive pay economies:

$$\frac{d\ln n}{d\ln z} = constant + \frac{1-\nu}{\nu} \cdot \mathbb{E}\left[\frac{1}{1-\Lambda}\right]$$

Same Employment Response w/ Rigid Wage or Flexible Incentive Pay

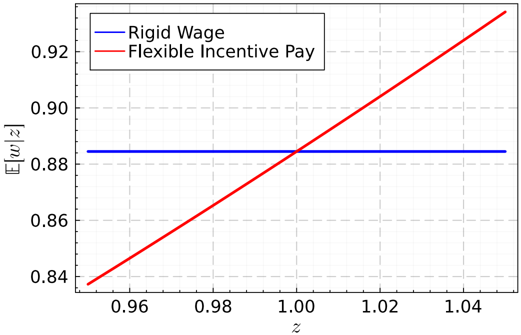
- **Fixed effort, fixed wages** (Hall)
  $\longrightarrow$ Large fluctuations in n when z fluctuates

- **Incentive contract**
  $\longrightarrow$ 1st order identical to rigid wage economy!

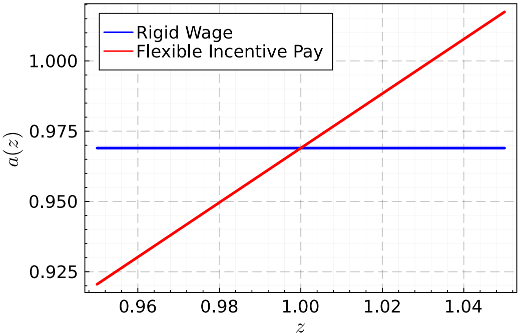


Parameterization | Full Information Benchmark | Proof Outline

Holds even though average wages can be strongly "pro-cyclical"



$\mathbb{E}_\eta[w^*(z,y)|z]$



$a^*(z)$

Result #1: wage cyclicality from incentives does **not** dampen unemployment dynamics

- NB: Output dynamics not equivalent

▸ Proof Outline ▸ Envelope Intuition ▸ What is a Bonus?

Roadmap

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Introducing Sticky Prices: Model Preliminaries

Final Goods Producer

$$Y = \left(\int_0^1 Y_j^{\frac{\alpha-1}{\alpha}} \right)^{\frac{\alpha}{\alpha-1}} \quad \Longrightarrow \quad P = \left(\int_0^1 p_j^{1-\alpha} \right)^{\frac{1}{1-\alpha}}$$

Retailers and Price Setting

$$\max_{p_j, Y_j H_j} p_j(Y_j) Y_j - zH_j \quad s.t. \quad Y_j = AH_j$$

Optimal Price

$$p_j^* = \mu \cdot z/A$$

Labor Market & Wholesale goods

- Wholesalers hire labor in frictional labor market as above, and sell at price z

Calvo Friction

- In middle of period, before output produced, there is a shock to real marginal cost z/A
- Calvo friction: a fraction ϱ of retailers can adjust their price and fully passthrough shock to prices

Incentive Pay Does Not Affect Slope of Phillips Curve

- Change in price level between beginning and end of period is:

$$\Pi = \varrho(d \ln z - d \ln A)$$

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- Previous: in both rigid wage and incentive pay economies

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- Phillips Curve relationship between inflation and market tightness/employment

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$\rightarrow$ Same SS Labor Share $\Longrightarrow$ **same slope of Phillips Curve** in both rigid and incentive wage economies

- **Intuition:** Marginal costs are rigid with optimal incentive pay despite cyclical wages

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Introducing Bargaining & Outside Option Fluctuations

- Allow for reduced form "bargaining rule" $\mathcal{B}(z)$ (Michaillat 2012):

$$J(z) \equiv \max_{a(z),w(z,y)} \mathbb{E}[z(a(z)+\eta) - w(z,y)]$$

subject to

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- Properties of the contract:
  1. Bargaining or cyclical outside option $\implies \mathcal{B}'(z) > 0$
  2. Wages can be cyclical either from incentives or because $B'(z) > 0$

Wage Cyclicality from Bargaining Does Dampen Unemployment Responses

Result #3: Wage cyclicality from bargaining or outside option does dampen unemployment dynamics

$$\frac{dJ}{dz} = a^* - \lambda^* \mathcal{B}'(z)$$

- Direct productivity effect a^*
- Cyclical utility from bargaining or outside option $\mathcal{B}'(z)$
- λ^* = Lagrange multiplier on participation constraint

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$$\lambda^* \mathcal{B}'(z) = \underbrace{\mathbb{E}\left[\frac{dw^*}{dz} - z\frac{da^*}{dz}\right]}_{\text{non-incentive wage cyclicality}}$$

- Direct productivity effect a^*
- Cyclical utility from bargaining or outside option $\mathcal{B}'(z)$
- λ^* = Lagrange multiplier on participation constraint
- $\lambda^* \mathcal{B}'(z)$ is non-incentive wage cyclicality

Intuition: higher wages from bargaining or outside option not accompanied by higher effort

$\Longrightarrow$ Marginal costs cyclical: same mechanism as standard model (e.g. Shimer 2005)

▸ Lagrangian

Static Model

Dynamic Model

Numerical Exercise

Conclusion

Summary of Dynamic Model

Diamond-Mortensen-Pissarides labor market

- Firms post vacancies, match with unemployed in frictional labor market w/ tightness θ_t
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Dynamic incentive contract (Sannikov 2008)

- General production and utility functions $f(z_t, \eta_t)$ and $u(w_t, a_t)$, discount factor β
- Unobservable history of effort a^t shifts distribution of observable persistent idiosyncratic shock η_t
- Firm offers dynamic incentive contract:

$$\left\{w_t\left(\eta^t, z^t\right), a_t\left(\eta^{t-1}, z^t\right)\right\}_{\eta^t, z^t, t=0}^{\infty}$$

1. Sequence of incentive constraints
2. Ex ante participation constraint w/ reduced form bargaining (ex ante promised utility $= \mathcal{B}(z_0)$)
3. Two sided commitment

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✓ Allows long term contracts (Barro 1977; Beaudry & DiNardo 1991) Details

Result#1: Incentive Wage Cyclicality Doesn't Mute Unemployment Fluct's

Shut down bargaining power + outside option $\rightarrow$ all wage cyclicality due to incentives

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Assume: (i) proximity to aggregate steady state (ii) production function is h.o.d. 1 in z, (iii) z is driftless random walk (iv) no worker bargaining power + constant outside option. In incentive pay economy

$$d\log\theta_0 \propto \left(\frac{1}{1 - \textit{labor share}}\right)\cdot d\log z_0, \qquad \textit{labor share} = \frac{\mathbb{E}_0[\textit{present value wages}]}{\mathbb{E}_0[\textit{present value output}]}$$

The same equations characterize a rigid wage economy with fixed wages + effort. ▸ *Expression*

Implication: incentive wage cyclicality does not mute unemployment responsiveness

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Proof sketch: optimal contract + envelope theorem

→ No first order effect of wage + effort changes on profits in response to z_0

→ Same profit response as if fixed wages + effort

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Generality: analytical results with general functions, persistent idiosyncratic shocks ▸ Assumptions

- *In paper:* same result w/ efficient endogenous separations

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Result in paper: bargained wage cyclicality **does** mute unemployment responsiveness ▸ Details

Result # 2: Slope of Price Phillips Curve Unaffected by Incentive Pay

- Same set-up as static model ▸ Details
  - Labor $\longrightarrow$ wholesalers $\longrightarrow$ sticky price retailers $\longrightarrow$ final goods producer

- Price Phillips Curve from linearized Calvo pricing problem

$$\Pi_t = \beta \mathbb{E}_t \Pi_{t+1} + \vartheta \zeta^{-1} \left(\ln \theta_t - \ln \bar{\theta} \right) - \vartheta \ln A_t$$

where $\vartheta \equiv (1-\varrho)(1-\beta\varrho)/\varrho$ and $\zeta \equiv d \ln \theta / d \ln z$ summarize nominal and real rigidity, respectively

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- $d \ln \theta / d \ln z$ equal near steady state in both rigid wage and incentive pay economies $\Rightarrow$ same PC

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- $d \ln \theta / d \ln z$ equal near steady state in both rigid wage and incentive pay economies $\Rightarrow$ same PC

- Also have equivalence in inflation-unemployment space

$$\Pi_t = \beta \mathbb{E}_t \Pi_{t+1} + \vartheta \tilde{\zeta} \left(u_t - \bar{u} \right) - \vartheta \ln A_t$$

with ϑ and $\tilde{\zeta}$ the same in rigid wage and incentive pay economies with same SS

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- Price Phillips Curve from linearized Calvo pricing problem

$$\Pi_t = \beta \mathbb{E}_t \Pi_{t+1} + \vartheta \zeta^{-1} \left(\ln \theta_t - \ln \bar{\theta} \right) - \vartheta \ln A_t$$

where $\vartheta \equiv (1-\varrho)(1-\beta\varrho)/\varrho$ and $\zeta \equiv d \ln \theta / d \ln z$ summarize nominal and real rigidity, respectively

- $d \ln \theta / d \ln z$ equal near steady state in both rigid wage and incentive pay economies $\Rightarrow$ same PC

- Also have equivalence in inflation-unemployment space

$$\Pi_t = \beta \mathbb{E}_t \Pi_{t+1} + \vartheta \tilde{\zeta} \left(u_t - \bar{u} \right) - \vartheta \ln A_t$$

with ϑ and $\tilde{\zeta}$ the same in rigid wage and incentive pay economies with same SS

- **Outstanding question:** how much of total wage cyclicality in data is due to incentives?

Static Model

Dynamic Model

Numerical Exercise

Conclusion

Numerical Exercise: Overview

Questions

- How much wage cyclicality due to incentives vs bargaining + outside option?
- How to calibrate simpler model of wage setting without incentives?

Approach

1. Explicit and tractable optimal contract building on Edmans et al (2012) Details
2. Reduced form bargaining: take-it-or-leave it with cyclical value of unemployment
3. Calibrate parameters targeting micro moments of wage adjustment

Heuristic Identification: Disentangling Bargaining from Incentives

1. **Ex post wage pass through informs incentives**
   - Key moments: pass-through of firm-specific profitability shocks to wages, variance of wage growth
   - Key parameter: disutility of effort, variance of idiosyncratic shocks
   - Conservative choices to reduce role of incentives (e.g. target low pass-through)
2. **Ex ante fluctuations in wage for new hires informs bargaining + outside option**
   - Key moment: new hire wage cyclicality
   - Key parameter: cyclicality of promised utility
3. **Externally calibrate standard parameters**
   - Separation rate, discount rate, vacancy cost, matching function (Petrosky-Nadeau and Zhang, 2017)
   - TFP process from Fernald (2014), accounting for capacity utilization of labor + capital

Optimal Contract Identification: Equations

Result#3: Substantial Share of Overall Wage Cyclicality Due to Incentives

Table: Data vs Simulated Model Moments

Moment	Description	Data	Baseline
$\text{std}(\Delta \log w_{it})$	Std. Dev. Log Wage Growth	0.064	0.064
$\partial \mathbb{E}[\log w_0]/\partial u$	New Hire Wage Cyclicality	-1.00	-1.00
$\partial \log w_{it}/\partial \log y_{it}$	Wage Passthrough: Firm Shocks	0.039	0.035
u_{ss}	SS Unemployment Rate	0.060	0.060
$\text{std}(\log u_t)$	Std. Dev. of unemployment rate	0.207	0.103
IWC	Share of Wage Cyclicality Due to Incentives	–	0.457

- Good match to targeted moments
- Rationalize about 1/2 of unemployment fluctuations in data
- **46% wage cyclicality due to incentives**

Parameter Estimates | Equivalence Plots | Calculating Bargained Share | Different Wage Target

User Guide: Calibrate Model w/o Incentives to Less Cyclical Wages

	Model: source of wage flexibility	
Moment	(1) Incentives + Bargaining	(2) No Incentives
$\partial \mathbb{E}[\log w_0]/\partial u$	-1.00	-0.54
$\partial \log \theta_0/\partial \log z_0$	13.6	13.3
$\text{std}(\log u_t)$	0.10	0.10

- Calibrate baseline model w/ bargaining + incentives and simple/standard model without incentives
- Analytical results suggest:
  - Calibrate bargaining + incentives model to overall wage cyclicality
  - Calibrate no-incentive model to non-incentive wage cyclicality **which is less procyclical**

User Guide: Calibrate Model w/o Incentives to Less Cyclical Wages

	Model: source of wage flexibility	
Moment	(1) Incentives + Bargaining	(2) No Incentives
$\partial \mathbb{E}[\log w_0]/\partial u$	-1.00	-0.54
$\partial \log \theta_0 / \partial \log z_0$	13.6	13.3
$\text{std}(\log u_t)$	0.10	0.10

- No incentive model calibrated to weakly cyclical wages
- Has similar employment dynamics to bargaining + incentives model w/ strongly cyclical wages

User Guide: Calibrate Model w/o Incentives to Less Cyclical Wages

	Model: source of wage flexibility	
	(1)	(2)
Moment	Incentives + Bargaining	No Incentives
$\partial \mathbb{E}[\log w_0]/\partial u$	-1.00	-0.54
$\partial \log \theta_0 / \partial \log z_0$	13.6	13.3
std($\log u_t$)	0.10	0.10

Takeaway:

- Can study simple models of wage setting without incentives
- But calibrate to relatively rigid wages

▸ All Wage Cyclicality from Bargaining ▸ IRFs

Static Model

Dynamic Model

Numerical Exercise

Conclusion

Conclusion

- *Does flexible incentive pay affect unemployment or inflation responses?*
- Incentive effect (effort moves) offsets wage effect so marginal costs are **rigid**

Results:

1. Incentive wage cyclicality **does not** dampen unemployment responses
2. Incentive wage cyclicality **does not** steepen slope of Phillips Curve
3. Non-Incentive wage cyclicality **does** dampen unemployment responses
    - Important to separately measure bargaining and incentives
    - Numerically: **46%** of wage cyclicality due to incentives
    - Calibrate simple model without incentives to weakly procyclical wages

Appendix

Why is employment log-linear in expected profits?

Free entry into vacancies

$$\kappa = q(v)J(z)$$

Substitute in for $q(v)$ and re-arrange for equilibrium vacancy posting

$$v^* = \left(\frac{\Psi J(z)}{\kappa}\right)^{\frac{1}{\nu}}$$

Now note that $n = f(v)$ (because initial unemployment $= 1$). Plug in to see

$$f(v) \equiv \frac{m(u,v)}{u} = \Psi v^{1-\nu} \qquad \Longrightarrow \qquad n = \left(\frac{\psi^{\nu+1}}{\kappa}\right)^{\frac{1}{\nu}} J(z)^{\frac{1-\nu}{\nu}}$$

Take logs to obtain result

$$\boxed{\ln n = constant + \left(\frac{1-\nu}{\nu}\right) \cdot \ln J(z)}$$

Technical Assumptions

Theorem

- The utility function u is Lipschitz continuous in the compact set of allocations
- z_t and η_t are Markov processes
- Local incentive constraints are globally incentive compatible
- The density $\pi\left(\eta_i^t, z^t | z_0, a_i^t\right)$ is continuous in the aggregate state z_0

Full Information Benchmark

◂ Employment Responses

- Firm observes aggregate productivity z and offers contract to worker
- Firm observes worker's effort a and idiosyncratic output shock η after production
- Firm offers contract to maximize profits

$$\max_{a(z,\eta),w(z,\eta)} J(z) = z\left(a(z,\eta)+\eta\right) - w(z,\eta)$$

subject to worker's participation constraint

$$\mathbb{E}_\eta\left[u\left(w(z,\eta),a(z,\eta)\right)\right] \geq \mathcal{B}$$

- First order condition implies optimal contract $a^*(z), w^*(z)$
- Yields fluctuations in profits

$$\frac{dJ(z)}{dz} = \mathbb{E}\left[a^*(z) + z\frac{da^*(z)}{dz} - \frac{dw^*(z)}{dz}\right] = a^*(z)$$

Parameterization

- CARA utility
$$u(c, a) = -e^{-r\left(c - \frac{\phi a^2}{2}\right)}$$
- Linear contracts
$$w(y) = \alpha + \beta y$$
  - α: "Base Pay"
  - β: "Piece-Rate" or "Bonus"
- Noise observed after worker's choice of action
- Yields optimal contract
$$\beta = \frac{z^2}{z^2 \phi r \sigma}, \qquad \alpha = b + \frac{\beta^2\left(\phi r \sigma^2 - z^2\right)}{2\phi}, \qquad a = \frac{\beta z}{\phi}$$

Static Model Parameter Values

- Elasticity of matching function $\nu = 0.72$ (Shimer 2005)
- Matching function efficiency $\psi = 0.9$ (Employment/Population Ratio = 0.6)
- Non-employment benefit $b = 0.2$ (Shimer 2005)
- Vacancy Creation Cost $\kappa = 0.213$ (Shimer 2005)
- CARA utility

$$u(c, a) = -e^{-r\left(c - \frac{\phi a^2}{2}\right)}$$

  with $\phi = 1$ and $r = 0.8$
- Linear contracts

$$w(y) = \alpha + \beta y$$

  - α: “Base Pay”
  - β: “Piece-Rate” or “Bonus”
- Profit shocks $\eta \sim \mathcal{N}(0, 0.2)$

- Frictional labor market: vacancy filling rate $q_t = \Psi \theta_t^{-\nu}$, market tightness $\theta_t \equiv v_t / u_t$

- Production function $y_{it} = f(z_t, \eta_{it})$
  - Density $\pi\left(\eta_i^t | z^t, a_i^t\right)$ of idiosyncratic shocks $\eta_i^t = \{\eta_{i0}, ..., \eta_{it}\}$
  - Affected by **unobservable** action $a_i^t = \{a_{i0}, ..., a_{it}\}$ + **observable** aggregate shocks z^t

- Dynamic incentive contract:
$\{\mathbf{w}, \mathbf{a}, \mathbf{c}, \mathbf{b}\} = \{w_{it}(\eta_i^t, z^t; z_0, b_{i0}), a_{it}(\eta_i^t, z^t; z_0, b_{i0}), c_{it}(\eta_i^t, z^t; z_0, b_{i0}), b_{i,t+1}(\eta_i^t, z^t; z_0, b_{i0})\}_{t=0,\eta_i^t,z^t}^{\infty}$

- Value of filled vacancy at time zero:

$$V \equiv \sum_{t=0}^{\infty} \int \int (\beta(1-s))^t \left(f(z_t, \eta_{it}) - w_{it}\left(\eta_i^t, z^t; z_0, b_{i0}\right)\right) \pi\left(\eta_i^t, z^t | z_0, b_{i0}, a_i^t\right) d\eta_i^t dz^t$$

s: exogenous separation rate, β: discount factor

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

$$[\text{PC}] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(c_{it}, a_{it}) + \beta s \mathcal{B}(b_{i,t+1}, z_{t+1}) \,|z_0, b_{i0}, a_i^t\right] = \mathcal{B}(b_{i0}, z_0)$$

$$\text{s.t.} \quad b_{i,t+1}(\eta_i^t, z^t) + c_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, b_{it}(\eta_i^t, z^t), \quad b_{it}(\eta_i^t, z^t) \geq \underline{b} \quad \text{assuming } r \text{ fixed}$$

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

$$[PC] \quad \sum_{t=0}^{\infty} (\beta (1-s))^t \, \mathbb{E}\left[u(c_{it}, a_{it}) + \beta s \mathcal{B}(b_{i,t+1}, z_{t+1}) \,|z_0, b_{i0}, a_i^t \right] = \mathcal{B}(b_{i0}, z_0)$$

s.t. $b_{i,t+1}(\eta_i^t, z^t) + c_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, b_{it}(\eta_i^t, z^t), \quad b_{it}(\eta_i^t, z^t) \geq \underline{b}$ assuming r fixed

- Incentive compatibility constraints: for all $\tilde{a}_i^t \in [\underline{a}, \overline{a}]^t, \tilde{c}_i^t \in [\underline{c}, \overline{c}]^t, \tilde{b}_i^{t+1} \geq [\underline{b}]^t$

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

$$[PC] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(c_{it}, a_{it}) + \beta s \mathcal{B}(b_{i,t+1}, z_{t+1}) \,|z_0, b_{i0}, a_i^t\right] = \mathcal{B}(b_{i0}, z_0)$$

s.t. $b_{i,t+1}(\eta_i^t, z^t) + c_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, b_{it}(\eta_i^t, z^t), \quad b_{it}(\eta_i^t, z^t) \geq \underline{b}$ assuming r fixed

- Incentive compatibility constraints: for all $\tilde{a}_i^t \in [\underline{a}, \overline{a}]^t$, $\tilde{c}_i^t \in [\underline{c}, \overline{c}]^t$, $\tilde{b}_i^{t+1} \geq [\underline{b}]^t$

$$[IC] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(\tilde{c}_{it}, \tilde{a}_{it}) + \beta s \mathcal{B}\left(\tilde{b}_{i,t+1}, z_{t+1}\right) |z_0, b_{i0}, \tilde{a}_i^t\right] \leq \mathcal{B}(b_{i0}, z_0)$$

s.t. $\tilde{b}_{i,t+1}(\eta_i^t, z^t) + \tilde{c}_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, \tilde{b}_{it}(\eta_i^t, z^t), \quad \tilde{b}_{it}(\eta_i^t, z^t) \geq \underline{b}$ assuming r fixed

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

$$[PC] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(c_{it}, a_{it}) + \beta s \mathcal{B}(b_{i,t+1}, z_{t+1}) \,|z_0, b_{i0}, a_i^t\right] = \mathcal{B}(b_{i0}, z_0)$$

s.t. $b_{i,t+1}(\eta_i^t, z^t) + c_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, b_{it}(\eta_i^t, z^t), \quad b_{it}(\eta_i^t, z^t) \geq \underline{b}$ assuming r fixed

- Incentive compatibility constraints: for all $\tilde{a}_i^t \in [\underline{a}, \overline{a}]^t$, $\tilde{c}_i^t \in [\underline{c}, \overline{c}]^t$, $\tilde{b}_i^{t+1} \geq [\underline{b}]^t$

$$[IC] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(\tilde{c}_{it}, \tilde{a}_{it}) + \beta s \mathcal{B}\left(\tilde{b}_{i,t+1}, z_{t+1}\right) |z_0, b_{i0}, \tilde{a}_i^t\right] \leq \mathcal{B}(b_{i0}, z_0)$$

s.t. $\tilde{b}_{i,t+1}(\eta_i^t, z^t) + \tilde{c}_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, \tilde{b}_{it}(\eta_i^t, z^t), \quad \tilde{b}_{it}(\eta_i^t, z^t) \geq \underline{b}$ assuming r fixed

- Loosely denote constraints as $PC(\mathbf{w}, \mathbf{a}, \mathbf{c}, \mathbf{b}; z_0, b_{i0}) = 0$, $IC(\mathbf{w}, \mathbf{a}, \mathbf{c}, \mathbf{b}; z_0, b_{i0}) \leq 0$

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

$$[\text{PC}] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(c_{it}, a_{it}) + \beta s \mathcal{B}(b_{i,t+1}, z_{t+1}) \,|z_0, b_{i0}, a_i^t\right] = \mathcal{B}(b_{i0}, z_0)$$

$$\text{s.t.} \quad b_{i,t+1}(\eta_i^t, z^t) + c_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, b_{it}(\eta_i^t, z^t), \quad b_{it}(\eta_i^t, z^t) \geq \underline{b} \quad \text{assuming } r \text{ fixed}$$

- Incentive compatibility constraints: for all $\tilde{a}_i^t \in [\underline{a}, \overline{a}]^t$, $\tilde{c}_i^t \in [\underline{c}, \overline{c}]^t$, $\tilde{b}_i^{t+1} \geq [\underline{b}]^t$

$$[\text{IC}] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(\tilde{c}_{it}, \tilde{a}_{it}) + \beta s \mathcal{B}\left(\tilde{b}_{i,t+1}, z_{t+1}\right) |z_0, b_{i0}, \tilde{a}_i^t\right] \leq \mathcal{B}(b_{i0}, z_0)$$

$$\text{s.t.} \quad \tilde{b}_{i,t+1}(\eta_i^t, z^t) + \tilde{c}_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, \tilde{b}_{it}(\eta_i^t, z^t), \quad \tilde{b}_{it}(\eta_i^t, z^t) \geq \underline{b} \quad \text{assuming } r \text{ fixed}$$

- Maximized value of a filled vacancy:

$$J(z_0, b_{i0}) \equiv \max_{\mathbf{w},\mathbf{a},\mathbf{c},\mathbf{b},\boldsymbol{\mu},\boldsymbol{\lambda}} \underbrace{V(\mathbf{w},\mathbf{a},\mathbf{c},\mathbf{b}; z_0, b_{i0})}_{\text{vacancy value}} + \underbrace{\langle \boldsymbol{\mu}, PC(\mathbf{w},\mathbf{a},\mathbf{c},\mathbf{b}; z_0, b_{i0})\rangle}_{\text{participation}} + \underbrace{\langle \boldsymbol{\lambda}, IC(\mathbf{w},\mathbf{a},\mathbf{c},\mathbf{b}; z_0, b_{i0})\rangle}_{\text{incentive compatibility}}$$

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

$$[\text{PC}] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(c_{it}, a_{it}) + \beta s \mathcal{B}(b_{i,t+1}, z_{t+1}) \,|z_0, b_{i0}, a_i^t\right] = \mathcal{B}(b_{i0}, z_0)$$

$$\text{s.t.} \quad b_{i,t+1}(\eta_i^t, z^t) + c_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, b_{it}(\eta_i^t, z^t), \quad b_{it}(\eta_i^t, z^t) \geq \underline{\text{b}} \quad \text{assuming } r \text{ fixed}$$

- Incentive compatibility constraints: for all $\tilde{a}_i^t \in [\underline{a}, \overline{a}]^t$, $\tilde{c}_i^t \in [\underline{c}, \overline{c}]^t$, $\tilde{b}_i^{t+1} \geq [\underline{b}]^t$

$$[\text{IC}] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(\tilde{c}_{it}, \tilde{a}_{it}) + \beta s \mathcal{B}\left(\tilde{b}_{i,t+1}, z_{t+1}\right) |z_0, b_{i0}, \tilde{a}_i^t\right] \leq \mathcal{B}(b_{i0}, z_0)$$

$$\text{s.t.} \quad \tilde{b}_{i,t+1}(\eta_i^t, z^t) + \tilde{c}_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, \tilde{b}_{it}(\eta_i^t, z^t), \quad \tilde{b}_{it}(\eta_i^t, z^t) \geq \underline{\text{b}} \quad \text{assuming } r \text{ fixed}$$

- Maximized value of a filled vacancy:

$$J(z_0, b_{i0}) \equiv \max_{\mathbf{w},\mathbf{a},\mathbf{c},\mathbf{b},\boldsymbol{\mu},\boldsymbol{\lambda}} \underbrace{V(\mathbf{w}, \mathbf{a}, \mathbf{c}, \mathbf{b}; z_0, b_{i0})}_{\text{vacancy value}} + \underbrace{\langle \boldsymbol{\mu}, PC(\mathbf{w}, \mathbf{a}, \mathbf{c}, \mathbf{b}; z_0, b_{i0}) \rangle}_{\text{participation}} + \underbrace{\langle \boldsymbol{\lambda}, IC(\mathbf{w}, \mathbf{a}, \mathbf{c}, \mathbf{b}; z_0, b_{i0}) \rangle}_{\text{incentive compatibility}}$$

- Free entry condition pins down market tightness: $\mathbb{E}_b[J(z_0, b_{i0})] = \frac{\kappa}{q(\theta_0)}$

Static Model Proof Outline

- Firm's value given by Lagrangian

$$J(z) = \mathbb{E}[z(a^*(z) + \eta) - w^*(z, y)] + \lambda \cdot (\mathbb{E}\left[u(w^*(z, y), a^*(z))\right] - \mathcal{B}) + \mu \cdot [IC]$$

  for λ and μ Lagrange multipliers on PC and IC, respectively.
- Take derivative w.r.t. z

$$\frac{dJ}{dz} = \mathbb{E}[a^*(z)] + z\frac{d\mathbb{E}[a^*(z, y)]}{dz} - \frac{d\mathbb{E}[w^*(z, y)]}{dz} + [PC] \cdot \frac{d\lambda}{dz} + [IC] \cdot \frac{d\mu}{dz} + \lambda\frac{\partial PC}{\partial z} + \mu\frac{\partial IC}{\partial z}$$

- Blue terms sum to zero by envelope theorem
- Red terms equal to zero as z does not appear in them
- Thus only direct term left

Intuition for Envelope Result

- Firm is trading off incentive provision and insurance
- Suppose z rises $\Rightarrow$ changes desired effort
- If z and a complements (as here), increase desired effort
- Incentivize worker $\Rightarrow$ steeper output-earnings schedule $\Rightarrow$ expose worker to more risk
- Must pay worker more in expectation to compensate for more risk
- Mean wage and effort move together
- Optimal contract $\Rightarrow$ marginal incentive and insurance motives offset

Aside: Interpretation of Bonus vs. Base Pay in Incentive Model

What is a bonus payment?

- Incentive contract is $w^* (\eta)$ = mapping from idiosyncratic shocks to wages
- Base wage = "typical" value of $w^* (\eta)$
- Bonus wage = $w^* (\eta)$ – base wage

Example 1: two values of idiosyncratic shock $\eta \in \{\eta_L, \eta_H\}$

- Base = $\min_\eta w(\eta)$, Bonus = $w(\eta)-$Base

Example 2: continuous distribution of η

- Base = $\mathbb{E}_\eta[w(\eta)]$, Bonus = $w(\eta)-$Base

$\rightarrow$ Specific form will depend on context but does not affect equivalence results

Isomorphism of Bargaining to TIOLI w/ cyclical unemp. benefit

Suppose worker and firm Nash bargain over promised utility $\mathcal{B}$ when meet

$$\mathcal{B}(z) \equiv arg \max_E J(z, E)^\phi \cdot (E - U(z))^{1-\phi}$$

Key: firm profits still determine employment fluctuations and defined as

$$\begin{aligned} J(z, \mathcal{B}) &= \max_{\mathbf{a},\mathbf{w}} EPDV(\text{Profits}) \\ s.t. \quad & \mathbf{a} \text{ is incentive compatible} \\ & \text{Worker's expected utility under contract } \geq \mathcal{B} \end{aligned}$$

Under TIOLI contract offers, $\mathcal{B}(z) = U(z)$ so that

$$\mathcal{B}(z) = U(z) = b(z) + \beta \mathbb{E}[\mathcal{B}(z')|z]$$

whether $\mathcal{B}(z)$ moves due to bargaining or $b(z)$ moves is first-order irrelevant to $J(z)$ and thus unemployment

Optimal Contract

Expressions Environment ID: Main Slides ID: Equations

- Wages are a random walk

$$\ln w_{it} = \ln w_{it-1} + \psi h'(a_t) \cdot \eta - \frac{1}{2}(\sigma_\eta h'(a_t))^2$$

  initialized at

$$w_{-1}(z_0) = \psi \left(Y(z_0) - \frac{\kappa}{q(\theta_0)} \right)$$

  for $\psi \equiv (\beta(1-s))^{-1}$ dubbed the "pass-through parameter" and $Y(z_0)$ the EPDV of output
- Effort increasing in z_t and satisfies

$$a_t(z_t) = \left[\frac{z_t a_t(z_t)}{\psi \left(Y(z_0) - \frac{\kappa}{q(\theta_0)} \right)} - \frac{\psi}{\varepsilon} (h'(a_t)\sigma_\eta)^2 \right]^{\frac{\varepsilon}{1+\varepsilon}}$$

- Worker utility under the contract equals $\mathcal{B}(z_0)$, the EPDV of unemployment utility
  - Cyclical $b(z) \implies w_{-1}(z)$ cyclical so influence new hire wages

Quantitative Contract: More Expressions

- EPDV of output

$$Y(z_0) \equiv \sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}\left[z_t(a_t+\eta_t)|z_0\right]$$

- Worker utility under contract

$$\frac{\log w_{-1}}{\psi} - \mathbb{E}_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^{t-1}\left(\frac{\psi}{2}(h'(a_t)\sigma_\eta)^2 + h(a_t) + \beta s \mathcal{B}(z_{t+1})\right)|z_0\right] = \underbrace{\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \ln b(z_t)|z_0\right]}_{\mathcal{B}(z_0)}.$$

Identification: Some Equations ▸ Optimal Contract ◂

Variance of log wage growth is

$$Var(\Delta \ln w_t) = \psi^2 Var(h'(a)\eta) \approx (\psi h'(a))^2 \sigma_\eta^2$$

Pass through of idiosyncratic firm output shocks to wages is

$$\frac{d \ln w_{it}}{d \ln y_{it}} = \frac{d \ln w}{d\eta} \cdot \left(\frac{d \ln y}{d\eta}\right)^{-1} = \psi h'(a) \cdot \left(\frac{1}{a+\eta}\right)^{-1}$$

Wages martingale $\implies$ new hire wages equal to w_{-1}/ψ in expectation, and $\ln w_{-1}$ equal to outside option:

$$\frac{\log w_{-1}}{\psi} - \mathbb{E}_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^{t-1}\left(\frac{\psi}{2}(h'(a_t)\sigma_\eta)^2 + h(a_t) + \beta s \mathcal{B}(z_{t+1})\right)|z_0\right] = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t(\ln\gamma + \chi \ln z_t)|z_0\right]$$

Differentiating both sides w.r.t. z shows clear relationship between χ (RHS) and $d \ln w_{-1}/d \ln z_0$

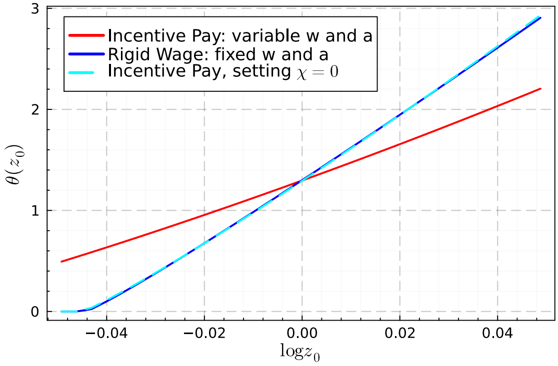
Externally Calibrated Parameters

Parameter	Description	Value	Source
β	Discount Factor	$0.99^{(1/3)}$	Petrosky-Nadeau & Zhang (2017)
s	Separation Rate	0.031	Re-computed, following Shimer (2005)
κ	Vacancy Cost	0.45	Petrosky-Nadeau & Zhang (2017)
ι	Matching Function	0.8	Petrosky-Nadeau & Zhang (2017)
ρ_z	Persistence of z	0.966	Fernald (2012)
σ_z	S.D. of z shocks	0.0056	Fernald (2012)

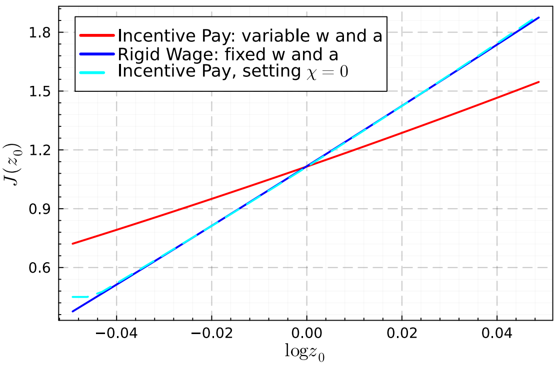
Estimated Parameters

Parameter	Description	Estimate	Bargain Estimate
σ_η	Std. Dev. of Noise	0.52	0^*
χ	Elasticity of unemp. benefit to cycle	0.49	0.63
γ	Steady State unemp. benefit	0.43	0.48
ε	Effort Disutility Elasticity	3.9	1^*

Equivalence Theorem Numerically



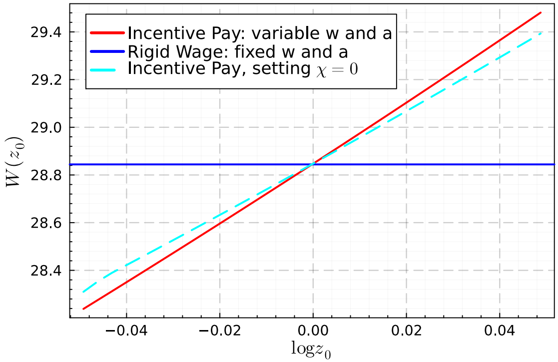
Panel A: Tightness



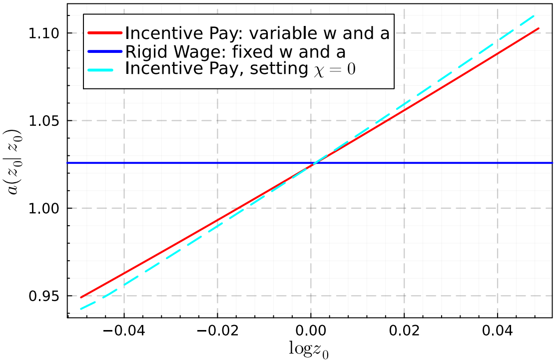
Panel B: Expected profits

- Observe equivalence between incentive pay economy setting $\chi = 0$ (light blue) and rigid wage/effort (dark blue) economies

Wage Differences: Full model vs Incentives Only



Panel A: EPDV of wages w_{-1}



Panel B: Effort of New Hires

- Removing bargaining reduces slope of wage-productivity schedule

Calculating Share of Wage Cyclicality due to Bargaining

1. Calculate total profit cyclicality in full model $\frac{dJ}{dz}$
2. Calculate direct productivity effect

$$(\mathbf{A}) = \sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}_0 f_z(z_t, \eta_{it}) \frac{\partial z_t}{\partial z_0}$$

3. Calculate "($\mathbf{C}$) term" as difference between profit cyclicality and direct productivity effect

$$(\mathbf{C}) = \frac{dJ}{dz} - (\mathbf{A})$$

4. Bargained wage cyclicality share is share of profit fluctuations due to (C) term

$$BWS = -\frac{(\mathbf{C})}{dJ/\,dz}$$

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

$$[\text{PC}] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(w_{it}, a_{it}) + \beta s \mathcal{B}(z_{t+1}) \,|z_0, a_i^t\right] = \mathcal{B}(z_0)$$

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- Incentive compatibility constraints: for all $\left\{\tilde{a}\left(\eta_i^{t-1}, z^t; z_0\right)\right\}_{t=0,\eta_i^t,z^t}^{\infty}$

$$[IC] \quad \sum_{t=0}^{\infty} (\beta (1-s))^t \, \mathbb{E}\left[u\left(w_{it}, \tilde{a}_{it}\right) + \beta s \mathcal{B}\left(z_{t+1}\right) | z_0, \tilde{a}_i^t\right] \leq \mathcal{B}\left(z_0\right)$$

- Loosely denote constraints as $PC(\mathbf{w}, \mathbf{a}; z_0) = 0$, $IC(\mathbf{w}, \mathbf{a}; z_0) \leq 0$

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- Maximized value of a filled vacancy:

$$J(z_0) \equiv \max_{\mathbf{w},\mathbf{a},\boldsymbol{\mu},\boldsymbol{\lambda}} \underbrace{V(\mathbf{w},\mathbf{a};z_0)}_{\text{vacancy value}} + \underbrace{\mu PC(\mathbf{w},\mathbf{a};z_0)}_{\text{participation}} + \underbrace{\langle \boldsymbol{\lambda}, IC(\mathbf{w},\mathbf{a};z_0)\rangle}_{\text{incentive compatibility}}$$

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- Free entry condition pins down market tightness: $J(z_0) = \frac{\kappa}{q(\theta_0)}$

A Dynamic Incentive Contract Equivalence Theorem

Assume (i) local constraints are globally incentive compatible (ii) unemployment benefits b are constant.

Technical Assumptions

The elasticity of market tightness with respect to aggregate shocks is to a first order

$$\frac{d\log\theta_0}{d\log z_0}=\frac{1}{\nu}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t E_{0,a^*}f_z\left(z_t,\eta_{it}\right)\frac{\partial z_t}{\partial z_0}z_0}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\left(E_{0,a^*}f\left(z_t,\eta_{it}\right)-E_{0,a^*}w_{it}^*\right)},$$

where a_{it}^* and w_{it}^* are effort and wages under the firm's optimal incentive pay contract.

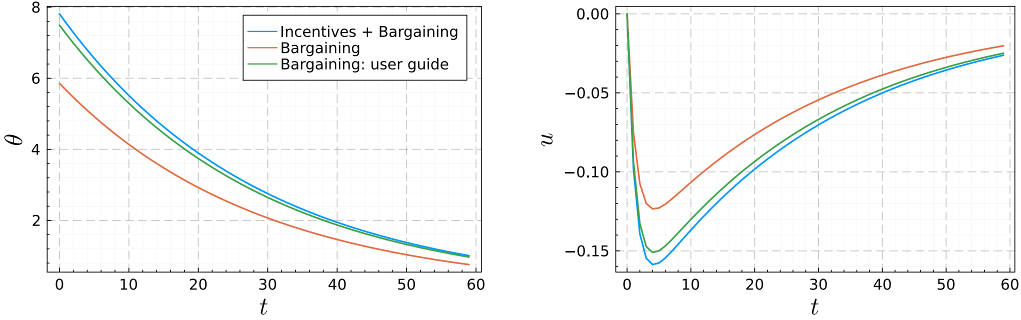
The elasticity of market tightness in a rigid wage economy with $w=\bar{w}$ and $a=\bar{a}$ is

$$\frac{d\log\theta_0}{d\log z_0}=\frac{1}{\nu}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t E_{0,\bar{a}}f_z\left(z_t,\eta_{it}\right)\frac{\partial z_t}{\partial z_0}z_0}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\left(E_{0,\bar{a}}f\left(z_t,\eta_{it}\right)-E_0\bar{w}\right)}.$$

Equivalence in Richer Models

- **Private savings and borrowing constraints** (Aiyagari 1993; Krusell et al 2010)
  - Equivalent impact elasticities
- **Endogenous separations** (Mortensen & Pissarides 1994)
  - Equivalent impact elasticities

Impulse Response to a 1SD shock to z_0



Panel A: Tightness θ_t Panel B: Unemployment u_t

TFP Shock

Numerical Results: Internally Calibrating Productivity Process

		Model: source of wage flexibility	
		(1)	(2)
Moment	Data	Incentives + Bargaining	Bargaining
ρ_y	0.89	0.89	0.89
σ_y	0.02	0.02	0.02
$\text{std}(\ln u_t)$	0.20	0.07	0.09
$d \ln \theta_0 / d \ln z_0$	-	18.7	11.6
$\mathcal{W}_0 / \mathcal{Y}_0$	-	0.96	0.96
$d \ln \mathcal{W}_0 / d \ln z_0$	-	0.55	0.37
$d \ln \mathcal{Y}_0 / d \ln z_0$	-	0.92	0.61
Incentive share	-	0.40	0.00

Numerical Results: Varying New Hire Wage Target

	Model: $\partial\mathbb{E}[\ln w_0]/du$ target			
Moment	-0.50	-0.75	-1.25	-1.50
$d\mathbb{E}[\ln w_0]/du$	-0.50	-0.75	-1.25	-1.50
std($\ln u_t$)	0.16	0.13	0.09	0.08
$d\ln\theta_0/d\ln z_0$	17.9	15.8	12.0	10.5
Incentive Wage Cyclicality share	0.73	0.59	0.38	0.33
Incentive Wage Cyclicality	-0.37	-0.44	-0.47	-0.49

Illustration: Wage Cyclicality and Unemployment Responsiveness

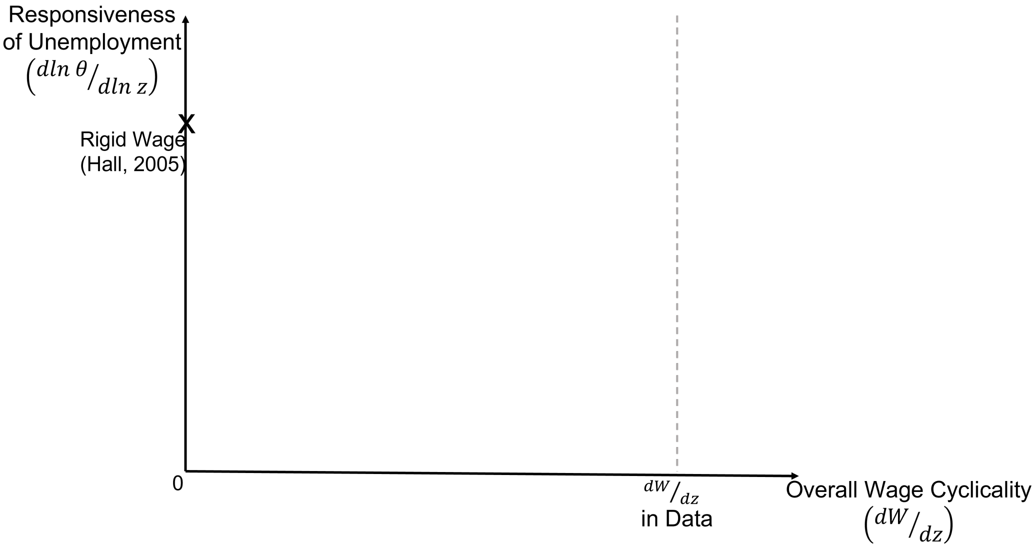
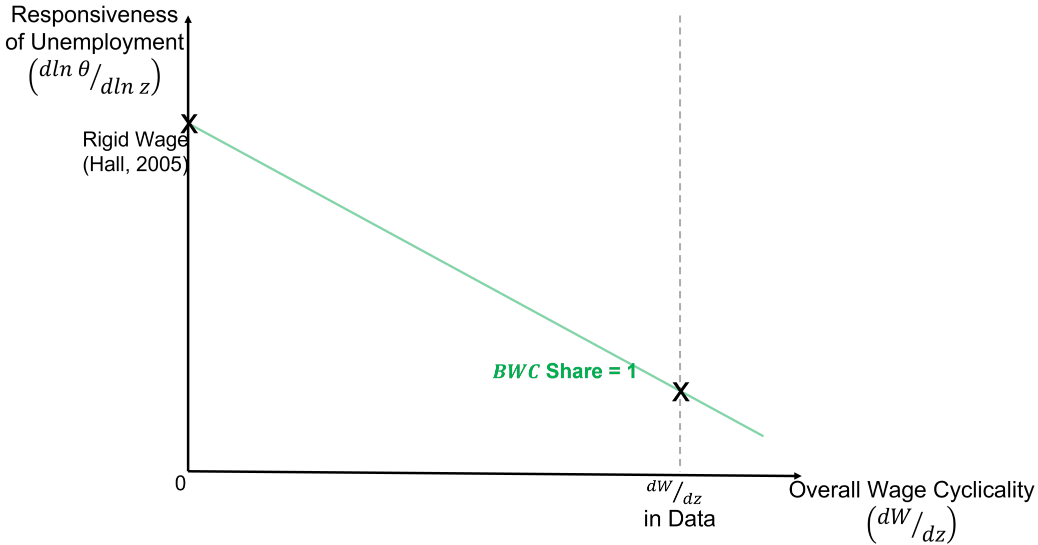


Illustration: Wage Cyclicality and Unemployment Responsiveness



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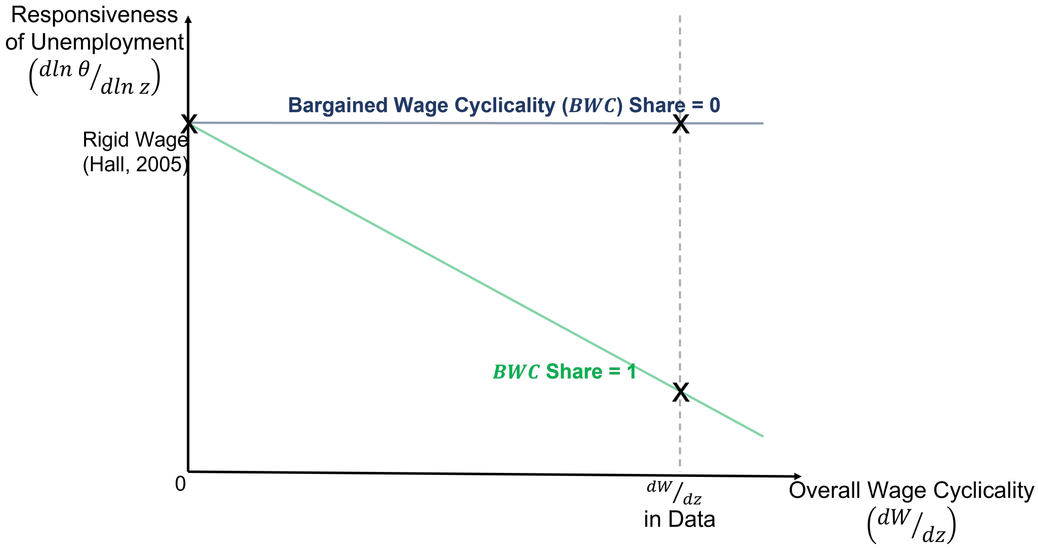

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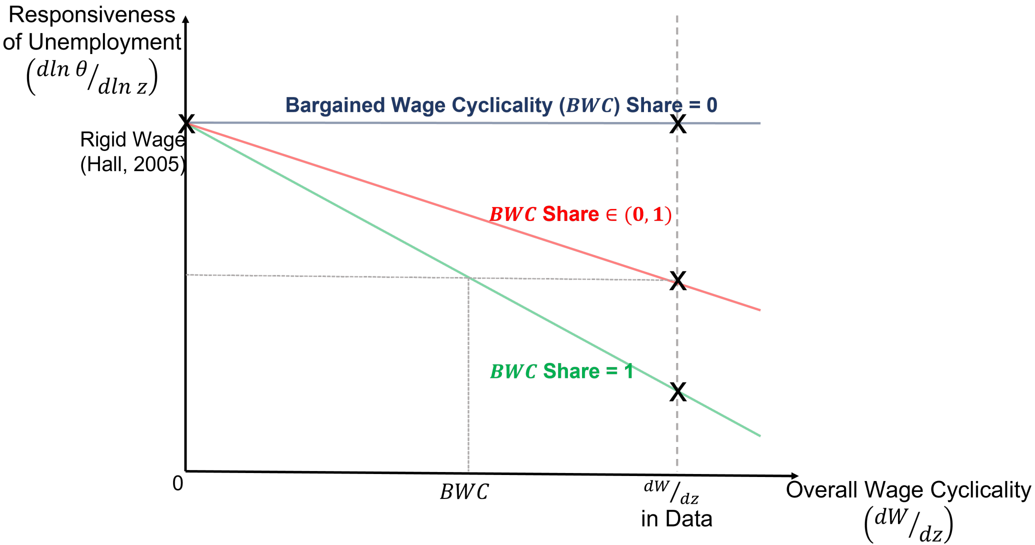

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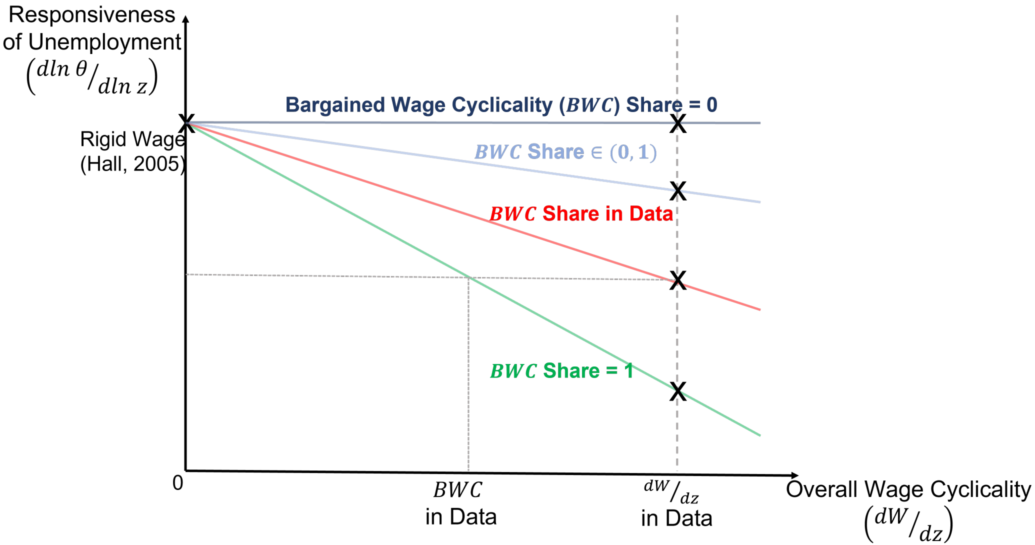
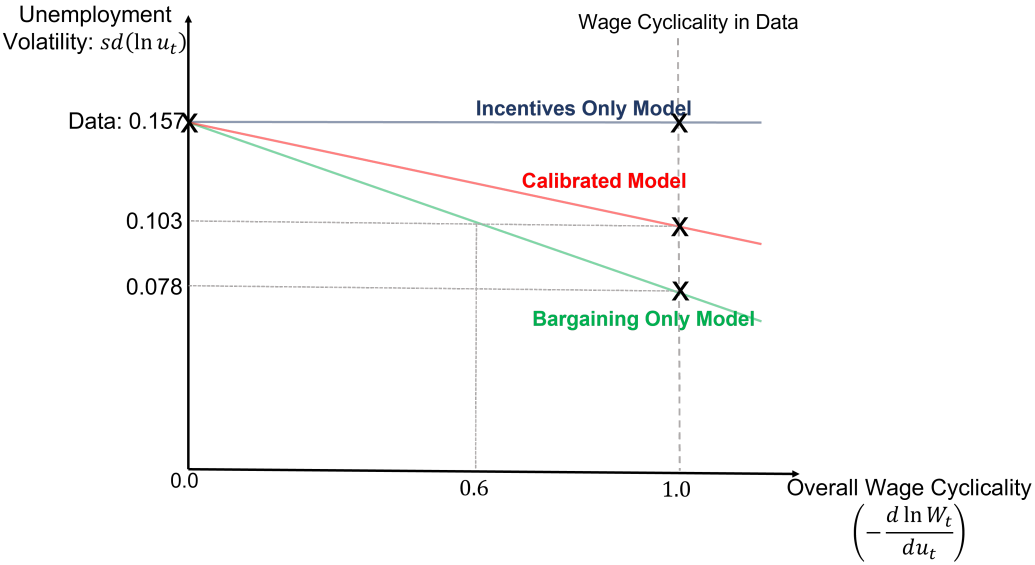


Quantitative Results: Graphical Illustration



Literature

- **Empirics of wage adjustment.** Devereux 2001; Swanson 2007; Shin & Solon 2007; Carneiro et al 2012; Le Bihan et al 2012; Haefke et al 2013; Kudlyak 2014; Sigurdsson & Sigurdardottir 2016; Kurmann & McEntarfer 2019; Grigsby et al 2021; Schaefer & Singleton 2022; Hazell & Taska 2022; Bils et al. 2023

  Contribution: model of wage setting consistent with micro evidence on bonuses

- **Wage adjustment and unemployment dynamics.** Shimer 2005; Hall 2005; Gertler & Trigari 2009; Christiano et al 2005; Gertler et al 2009; Trigari 2009; Christiano et al 2016; Gertler et al 2020; Blanco et al 2022

  Contribution: Flexible incentive pay does not dampen unemployment fluctuations

- **Incentive contracts.** Holmstrom 1979; Holmstrom & Milgrom 1987; Sannikov 2008; Edmans et al 2012; Doligalski et al. 2023

  Contribution: Characterize aggregate dynamics with general assumptions
  (E.g. non-separable utility, persistent idiosyncratic shocks, no reliance on "first order approach")

- **Sales + Rigidity.** e.g. Nakamura & Steinsson 2008; Klenow & Kryvtsov 2008; Kehoe & Midrigan 2008; Eichenbaum et al 2011

  Contribution: incentive pay does not affect aggregate rigidity even if bonuses are cyclical

- Frictional labor market: vacancy filling rate $q_t \equiv q(\theta_t)$, market tightness $\theta_t \equiv v_t / u_t$
- Production function $y_{it} = f\left(z_t, \eta_{it}\right)$
  - Density $\pi\left(\eta_i^t | a_i^t\right)$ of idiosyncratic shocks $\eta_i^t = \{\eta_{i0}, ..., \eta_{it}\}$
  - Affected by **unobservable** action $a_i^t = \{a_{i0}, ..., a_{it}\}$, $a_{it} \in [\underline{a}, \overline{a}]$
- Dynamic incentive contract: $\{\mathbf{a}, \mathbf{w}\} = \left\{a\left(\eta_i^{t-1}, z^t; z_0\right), w\left(\eta_i^t, z^t; z_0\right)\right\}_{t=0, \eta_i^t, z^t}^{\infty}$
- Value of filled vacancy at time zero:

$$V(\mathbf{a}, \mathbf{w}; z_0) \equiv \sum_{t=0}^{\infty} \int \int \left(\beta\left(1-s\right)\right)^t \left(f\left(z_t, \eta_{it}\right) - w_{it}\left(\eta_i^t, z^t; z_0\right)\right) \pi\left(\eta_i^t, z^t | a_i^t\right) d\eta_i^t dz^t$$

s: exogenous separation rate, β: discount factor

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

$$[PC] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(w_{it}, a_{it}) + \beta s U(z_{t+1}) \,|z_0, a_i^t\right] = \mathcal{B}(z_0)$$

  - "Reduced form" **bargaining power** if $\mathcal{B}'(z_0) > 0$
  - Formulation of bargaining power nests e.g. Nash w/ cyclical outside option, Hall-Milgrom bargaining

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$$[PC] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(w_{it}, a_{it}) + \beta s U(z_{t+1}) \,|z_0, a_i^t\right] = \mathcal{B}(z_0)$$

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- Free entry condition pins down market tightness: $J(z_0) = \frac{\kappa}{q(\theta_0)}$

Incentive Wage Cyclicality Doesn't Mute Unemployment Fluct's

Temporarily shut down bargaining power $\rightarrow$ all wage cyclicality is due to incentives

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Assume: (i) proximity to non-stochastic steady state (ii) production function is h.o.d. 1 in z, (iii) **contracts offer constant promised utility $\mathcal{B}$.** *Then in the flexible incentive pay*

$$\frac{d\log\theta_0}{d\log z_0} = \frac{1}{\nu_0}\frac{1}{1 - \textit{labor share}}$$

where

$$\textit{labor share} = \frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^{t} E_0 w_{it}}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^{t} E_{0,a} f\left(z_0, \eta_{it}\right)}$$

The same equations characterize a rigid wage economy with $w_{it} = \bar{w}, a_{it} = \bar{a}$

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The same equations characterize a rigid wage economy with $w_{it} = \bar{w}, a_{it} = \bar{a}$

Implications: incentive wage cyclicality does not mute unemployment fluctuations

- In an incentive pay economy with **flexible** dynamic incentive pay
- Unemployment dynamics behave "as if" wages are **rigid**

Parameterized Dynamic Incentive Contract Model

- Linear production
- Normally distributed noise $\eta \sim \mathcal{N}(0, \sigma_\eta)$, agg. productivity AR(1) in logs
- Log and isolastic utility

$$u(c, a) = \ln c - \frac{a^{1+1/\varepsilon}}{1 + 1/\varepsilon}$$

- Agent observes η before deciding action
- Worker's flow consumption during unemployment is $b(z) \equiv \gamma z^{\chi}$
- Firm makes take-it-or-leave-it offers to worker so

$$\mathcal{B}(z_0) = \sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[\ln \gamma + \chi \ln z_t | z_t\right]$$

  - First-order equivalent to fixed b and bargaining over surplus
  - χ governs cyclicality of promised utility and thus "bargained wage cyclicality"

Optimal Contract

Regularity Conditions

1. The distribution of innovations to aggregate productivity does not depend on initial productivity z_0

$$z_t = \mathbb{E}[z_t|z_0] + \varepsilon_t, \qquad \varepsilon^t \sim G_t(\varepsilon^t)$$

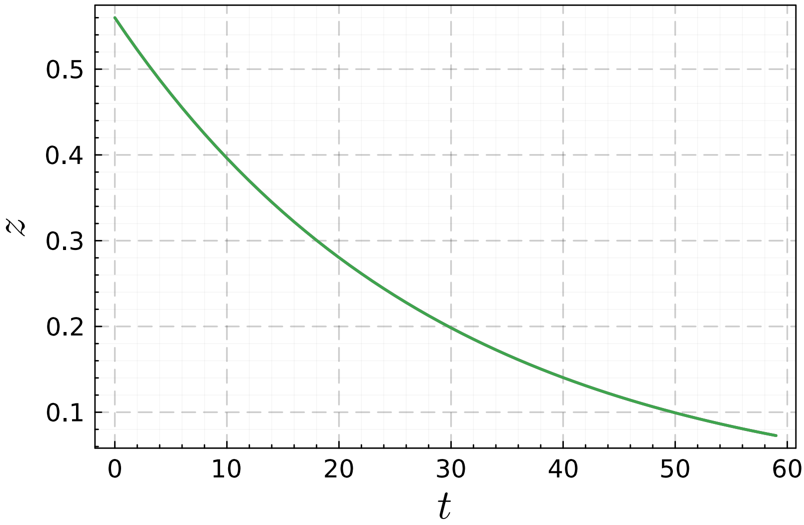
2. $f(z,\eta)$ is differentiable and strictly increasing in both of its arguments

3. $u(c,a)$ is strictly increasing and concave in c and decreasing and convex in a and is Lipschitz continuous

4. The set of feasible contracts that satisfy IC and PC is non-empty.

5. At least one of the following conditions holds
   - 5.1 The set of feasible contracts is convex and compact. The worker's optimal effort choice is fully determined by the first order conditions to their problem. Finally, idiosyncratic shocks η_{it} follow a Markov process: $\pi_t(\eta_t|\eta^{t-1}, a^t) = \pi_t(\eta_t|\eta_{t-1}, a_t)$
   - 5.2 Feasible contracts are continuous and twice differentiable in their arguments (z^t, η^t) with uniformly bounded first and second derivatives.

Exogenous TFP Shock



Bargained Wage Cyclicality: Dynamic Model Definition and Result

- Assume Inada conditions on utility and first-order Markov process for η.
- Define $\mathcal{Y}(\mathbf{a}^*(z_0), z_0)$ to be EPDV of match output given z_0
- Define $\mathcal{W}(z_0)$ to be EPDV of wage payments given z_0 under optimal contract
- Define Bargained Wage Cyclicality to be wage movements in excess of effort-induced output movements:
$$\frac{\partial W^{\text{bargained}}}{\partial z_0} = \frac{d\mathcal{W}(z_0)}{dz_0} - \partial_{\mathbf{a}}\mathcal{Y}(\mathbf{a}^*(z_0), z_0)\frac{d\mathbf{a}^*}{dz_0}$$
- Then
$$d \ln \theta_0 \propto \left(\frac{1 - BWC}{1 - \textit{labor share}}\right) d \ln z_0$$
where BWC is the share of overall wage cyclicality associated with bargaining, and $BWC > 0 \iff \mathcal{B}'(z_0) > 0$

Wage Cyclicality from Bargaining Does Dampen Unemployment Responses

Result #2: Wage cyclicality from bargaining or outside option does dampen unemployment dynamics

$$J(z) = \mathbb{E}[z(a(z) + \eta) - w(z, y)] + \lambda(z) \cdot IC + \mu(z) \cdot [\mathbb{E}[u(w, a)] - \mathcal{B}(z)]$$

- $\lambda(z)$ Lagrange multiplier on IC constraint
- $\mu(z)$ Lagrange multiplier on participation constraint

Wage Cyclicality from Bargaining Does Dampen Unemployment Responses

Result #2: Wage cyclicality from bargaining or outside option does dampen unemployment dynamics

$$\frac{dJ}{dz} = a^* - \mu^* \mathcal{B}'(z)$$

- Direct productivity effect a^*
- Cyclical utility from bargaining or outside option $\mathcal{B}'(z)$
- μ^* = Lagrange multiplier on participation constraint

Wage Cyclicality from Bargaining Does Dampen Unemployment Responses

Result #2: Wage cyclicality from bargaining or outside option does dampen unemployment dynamics

$$\frac{dJ}{dz} = a^* - \mu^* \mathcal{B}'(z) = a^* + \mathbb{E}\left[z\frac{da^*}{dz} - \frac{dw^*}{dz}\right]$$

$$\implies \underbrace{\mathbb{E}\left[\frac{dw^*}{dz} - z\frac{da^*}{dz}\right]}_{\text{bargained wage cyclicality}} = \mu^* \mathcal{B}'(z)$$

- Wages move in excess of effort if and only if $\mu^*(z)\mathcal{B}'(z) > 0$: cyclical ex-ante promised utility
- Dub $\mu^* \mathcal{B}'(z)$ bargained wage cyclicality

Intuition: higher wages from bargaining or outside option not accompanied by higher effort

- Same mechanism as standard model (e.g. Shimer 2005)

Dynamic Sticky Price Model Setup Details

- Unit measure of retailers j produce using wholesale good purchased at real price z_t:

$$Y_{jt} = A_t H_{jt}$$

- Retailers set prices at beginning of period as markup over expected marginal costs

$$p_{jt} = z_t / A_t$$

- An i.i.d. fraction ϱ of retailers can adjust their price each period
- Final output is Dixit-Stiglitz aggregate of retailers goods

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\alpha-1}{\alpha}} \right)^{\frac{\alpha}{\alpha-1}} \implies P_t = \left(\int_0^1 p_{jt}^{1-\alpha} \right)^{\frac{1}{1-\alpha}}$$

- Wholesalers hire labor in frictional labor market as above, and sell at price z_t